Forecasting Research

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October 1994

Meteorological Office
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[†] - UK Meteorological Office
[‡] - Hungarian Meteorological Service

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UK

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1. INTRODUCTION

Discussions at a World Meteorological Organisation workshop on Long-Range Prediction at Trieste during April 1991 resulted in a proposal that monthly forecasts might be tested for Hungary in a cooperative project using the Meteorological Office system. Funding for this project was subsequently obtained from the Commission of the European Communities (Contract No. ERBCIPACT 920290) to enable the third author to spend a period of three months (June to September 1993) in Bracknell undertaking development of the system. Additional funding was provided by the British Council and by the Hungarian National Committee for Scientific Development (OMFB) to support short return visits of the second author to Budapest (February 1994) and the third author to Bracknell (March 1994). Results from the first year of this collaborative programme are presented in this paper.

Long-range forecasting in Hungary has been based on empirical techniques for many years, the objective having been to provide predictions of monthly averages of temperature and rainfall out to 6 months (Csima *et al.*, 1991); a sample of two pages from an eight-page leaflet giving forecast details for May to October 1992 is provided as Figure 1. Although Csima *et al.* (1991) present some skill statistics for these forecasts there is some doubt over the validity of the assessment technique used, and thus part of the cooperative programme was to establish the skill of these forecasts using accepted methods. Monthly-average forecasts out to six months would, of course, seem inconsistent with current understanding of predictability theory, but the Hungarian Meteorological Service has been satisfying a customer requirement, and a customer perception that the predictions have value, in providing these forecasts.

Monthly forecasting for the United Kingdom originally developed along empirical lines throughout the 1960's to 1980's, including a period when analogues, equivalent to but not identical to those in Hungary, were used (Folland and Woodcock, 1986). Ensemble numerical forecasts, however, were introduced operationally to the United Kingdom forecasts during December 1988 (Milton, 1990), and most issued predictions are now based on the

numerical model products. Back-up to the ensembles is still provided by two empirical techniques, SPEVR and MVA, both based on eigenvectors of pressure fields, the former using multiple linear regression between fields and the latter linear discriminant analysis between predictor and predictand fields. The domain of SPEVR does not extend to Hungary, although Hungary is within the eastern boundary of the MVA domain. Because of the nature of the eigenvectors there is, however, minimal detail in MVA pressure fields around Hungary and therefore neither empirical technique has been adopted in the cooperative project. All experimental forecasts for Hungary have been based on ensemble predictions only.

Nine-member ensembles are produced every second week, with integrations covering the forecast period of 30 days. The lagged-average technique of Hoffmann and Kalnay (1983) is used, in which model runs are set off at each analysis cycle at 6-hour intervals. Thus the first run begins at 0000Z on the Saturday before the Monday forecast and the last at 0000Z on the Monday. Forecasts are examined at an expert conference at Bracknell and an appropriate period-averaged field selected as the basis of the deterministic forecasts for periods 6 to 15 and 16 to 30 days ahead (forecasts for 1 to 5 days are also made but using high-resolution medium-range operational model runs from either the Meteorological Office or the European Centre for Medium-Range Weather Forecasts - ECMWF). Monthly forecasts are constructed from those for the three sub periods. Normally the grand ensemble mean across all nine members is selected as the forecast base, theory indicating that this mean provides the most reliable estimate of the future atmospheric state. On occasion the mean of a sub cluster of like members might be chosen should other evidence, including that from the empirical techniques, support such a selection.

Linear specification equations are then applied to the selected fields to provide estimates of period-mean temperature anomalies from climatology (all of maxima, minima and averages - the mean of maxima and minima) and rainfall percentages of normal across ten climatological districts of the United Kingdom (Fig. 2). Temperature anomalies and rainfall percentages of normal are usually reduced to the appropriate quintile or tercile of the climatological

distributions respectively. The multiple-linear specification equations link temperature and rainfall to aspects of the pressure field, such as the gradient and the vorticity, surrounding each district. Additionally 1000 to 500 hPa thickness data are added to the equations for temperature (used only at days 1 to 5), as are sea-surface temperature anomalies upwind of each district. The resulting equations provide a simple method of inverting pressure fields regardless of whether these have been produced by empirical or numerical methods; recent examination of five-day forecasts achieved both by using all the facilities of the Central Forecast Office and by simple application of these equations to model output fields has revealed that skill levels achieved by the specification equations are highly competitive with those produced by more sophisticated methods. Historical issued United Kingdom forecast skills are significant at levels above 10%, particularly for days 1 to 5, 6 to 15 and 1 to 30, but there is no evidence of skill in rainfall predictions at days 16 to 30.

The objective of the cooperative programme with Hungary was to examine the levels of skill achievable by transporting the United Kingdom techniques to central Europe. Preliminary encouraging evidence of the possibility of achieving positive skills and useful forecasts in the east had been obtained from an independent analysis which demonstrated that model predictions were more skilful across eastern than over western Europe (Richardson, pers. comm.). In the first phase of the programme specification equations equivalent to those used in the United Kingdom were set up for Hungary; in the second the equations were applied to fields predicted by the ensemble in order to assess levels of skill achieved over Hungary and to contrast these with skills for the United Kingdom.

2. DEVELOPMENT OF THE SPECIFICATION EQUATIONS

Equations for Hungary were developed in a similar manner to those for the United Kingdom, except that for Hungary sea-surface temperature anomalies, naturally, were not used. Hungary was divided into five climatological areas (Fig. 3), which in part separated the mountainous north-east from the plains of the remainder of the country. Quality-controlled

data from about five stations per district were available from 1961 up to 1992 (for the United Kingdom there are about 15 stations per district); data for the period up to 1990 were used in development of the equations, with those for the final two years retained for independent assessment of equation skill (current United Kingdom equations were established across the period 1951 to 1980). Equations were developed for three averaging periods, of 5, 10 and 15 days, to match the forecast periods. For any given month, temperature, rainfall and atmospheric circulation data were averaged across all possible periods within the month (e.g. into six 5-day periods etc.); thus 15-day equations were developed using 60 values whereas 5-day equations used 180 values. By contrast United Kingdom equations, although originally developed for 5, 10 and 15-day periods but now limited only to 15-day periods, were created using data for each half-month together with those for the surrounding two half-months (90 training values).

Skill of the specification equations can be assessed in a number of ways, but that chosen here is through the Folland-Painting skill score (Folland *et al.*, 1986). This skill scores uses information theory to score categorical forecasts; in the case of temperature predictions quintiles of the distribution were used and for rainfall predictions terciles (all developed from data for 1961 to 1990). Scores of +100% are obtained for perfect categorical predictions whereas worst-possible forecasts score -100%; chance predictions, as well as climatology, are expected to score 0%. To place the results below in context, forecasts for the first five days typically score about 40 to 50% on the Folland-Painting scale. Equations for the United Kingdom appear more skilful than those for Hungary in a perfect-prognosis assessment, i.e. applying the specification equations to observed pressure fields (Fig. 4). In Figure 4 mean skills for 15-day periods are contrasted for the period 1961 to 1992 (to 1991 only for the United Kingdom) for equation sets including and excluding 1000 to 500 hPa thickness information. Although the contrast is not ideal because of the somewhat different assessment periods and the fact that only two years of data are independent for the Hungarian values as opposed to 11 years for those for the United Kingdom, it is apparent that:

a) temperature equations using thickness are more skilful than those without thickness (results for maximum and minimum temperatures are similar to those for average temperatures given in Figure 4);

b) addition of thickness information adds little to skill of the rainfall equations;

c) in general United Kingdom equations are more skilful than those for Hungary, particularly those for temperature without thickness and for rainfall.

The more limited skill of the equations over Hungary was anticipated. For temperature fields about 10% of the variance captured by the United Kingdom equations is contributed by the sea-surface temperature anomalies, information not applicable for land-bound Hungary. Otherwise, most of the temperature information is derived from pressure gradients (in effect the direction and strength of the geostrophic wind) and that for rainfall from the vorticity field. Because of their relative geographical locations, variations in both pressure gradients and vorticity fields are greater and are more closely linked to temperature and rainfall variations over the United Kingdom than across Hungary. The most extreme example is that of summer rainfall, for which skills of the Hungarian specification equations are relatively low. Most Hungarian summer rainfall originates in thunderstorms associated with systems which provide minimal impact on period-mean pressure fields; by contrast much summer rainfall in the United Kingdom is associated with well-defined disturbances in the westerlies. Similarly, the mainly anticyclonic summer circulation over Hungary is less likely to be closely related to temperature variations than is the circulation around the United Kingdom, although Hungarian temperature equations including thickness are encouraging in their levels of skill. Altogether the perfect-prognosis skills achieved with the equations are judged adequate to be tested in real-time predictions.

3. EXPERIMENTAL FORECASTS

Two methods of determining practical levels of skill achievable have been followed; in the first the specification equations have been applied to historical data from 25 January 1993

onwards, in the second to real-time data. In all cases scores obtained have been contrasted with those achieved over the United Kingdom from the same forecasts. Two forecasts from Bracknell have been used in the contrasts: the first is the issued prediction, i.e. the final result of all forecaster intervention (if any); the second an automated forecast based on inversion of either the operational forecast from Bracknell or ECMWF at days 1 to 5, as selected by the forecaster, or of the ensemble mean at longer range.

Real-time forecasts for Hungary started on 9 August 1993, the first three forecasts being made in Bracknell. Subsequently the essential model information has been sent by Email to Budapest, where the specification equations have been applied. Only recently has some form of forecast information display been available in Budapest (Fig. 3) and so forecasts have been treated in a non-interventionist manner, unlike the situation in the United Kingdom where forecasters are able to intervene at several stages in the forecast process. All Budapest forecasts have been based on the ensemble mean (except at days 1 to 5 when forecasts were based on the same operational model as used at Bracknell), although guidance of the forecast base selected in Bracknell was transmitted to Budapest. Occasionally a mean across a cluster of ensemble members is used at Bracknell; experience indicates that clusters appropriate to western Europe may not necessarily be appropriate in the east and so no clusters have been used in Budapest forecasts to date. Thus the automated United Kingdom forecast discussed above is the more directly comparable with the Hungarian forecast than is the United Kingdom issued forecast. Some attempts to use Kalman filtering in improving the outcome of the Hungarian specifications equations have been made, but with discouraging results.

Contrasting the skill of the Hungarian forecasts with those for the United Kingdom provides a useful comparison baseline, particularly as United Kingdom skills for much of the cooler months of 1993/94 have been well below long-term averages (temperature forecasts at 6 to 15 days in winter had particularly and uncharacteristically low skill). Occasional periods of limited skill are to be expected in long-range forecasting and, as the experimental Hungarian forecasts to date happened to have coincided with one such period, it is important to view the results in context. Overall skills for the two countries for the period since the first real-

time prediction on 9 August 1993 up to the forecast of 16 May 1994 are somewhat variable (Fig. 5), with Hungarian temperature forecasts perhaps the more skilful at days 1 to 5 and 6 to 15 against United Kingdom forecasts, although the relatively low skill of the Hungarian 1-to-5 day rainfall forecasts is surprising. No overall positive skill has been achieved for any of the Hungarian 16-to-30 day predictions, although, as mentioned above, United Kingdom skill was also limited for this period.

Hungarian seasonal rainfall skill scores at days 1 to 5 were substantially below those for issued United Kingdom forecasts during both autumn 1993 and winter 1993/94 (Fig. 6). The relatively poor overall score for rainfall (Fig. 5) resulted from unskilful predictions in these seasons, skill being present in spring 1994. Temperature scores at days 1 to 5, however, were positive in Hungary throughout, and normally exceeded those for both issued and automated United Kingdom forecasts. There is no clear pattern in scores for the two countries at either days 6 to 15 (Fig. 7) or days 16 to 30 (Fig. 8), although often negative skills in one country are accompanied by positive skills in the other, even when automated predictions are compared. Spring 1994 is the season with clearest opposition of signs between scores of automated forecasts for both countries. Generally higher skill was obtained over Hungary at both longer ranges during the period of experimental predictions than during that of hindcasts.

There appears to be more skill variability from forecast to forecast in the Hungarian than in the United Kingdom predictions (Figs. 9 to 11), although presently no reason can be offered to account for this. There is, perhaps, a general tendency for individual forecast skills to vary similarly in both countries, but this is by no means an absolute result and it is quite possible for a highly skilful prediction in one country to be accompanied by substantially negative skill in the other. More often than not Hungarian 1-to-5 day temperature predictions were more skilful than those for the United Kingdom, although the reverse is true for rainfall (Fig. 9). At days 6 to 15 (Fig. 10) and 16 to 30 (Fig. 11) there are periods extended across several forecasts for which positive skill in one country accompanies negative skill in the other; for example, compare the average temperature 6 to 15 day forecasts in summer 1993

and in winter 1993/94, for which roles reverse. At other times, such as winter 1993/94 and spring 1994 for 16-to-30 day average temperatures, skill sequences are similar in both countries.

4. SUMMARY AND CONCLUSIONS

A period of less than one year is insufficient to enable enough data to be collected for an adequate test of a long-range forecasting system; at the Meteorological Office skills are normally tested as averages across four-year periods. Further, the period over which Hungarian forecasts have been tested to date has been one during which United Kingdom skills were often below, sometimes well below, long-term averages. No hard conclusions can therefore be drawn from the data available other than to note that skills obtained for Hungary, both for temperatures and rainfall, appear approximately similar to those for the United Kingdom at all time ranges. Given that the perfect prognosis tests of the specification equations suggested that less skill was derived over Hungary, it might be tentatively suggested that the apparent equalisation of skills between the two countries may be related to the previously-noted higher quality of the model predictions over eastern than over western Europe.

Nevertheless the main objective of the CEC Project has been achieved in that a monthly forecasting facility based on ensemble products from the United Kingdom Meteorological Office has been developed for Hungary. The products are transferred to Budapest soon after they become available in Bracknell, and new automatic procedures in Budapest provide early access to the forecast information. The service of forecast products will continue to Budapest, and so ultimately more experimental data will become available to enable improved assessment of these forecasts.

In the continuing collaboration between Budapest and Bracknell further work is planned. One of the major changes to the manner in which forecasts will be provided in the United

Kingdom is that probabilistic, rather than the current deterministic, predictions will be made using information from all members of the ensemble rather than just from a mean across some or all members. Skill of probability forecasts for Hungary will also be examined. It is also planned to examine skills of model predictions across western and eastern Europe in more detail, and to compare skills from the ensemble system with those from the Hungarian analogue forecast system. Preliminary results from all of these studies are available but have not been detailed in this paper.

No decision has been taken as yet as to whether and when monthly forecasts using the ensemble system might be issued to users in Hungary, as currently happens in the United Kingdom. More data to confirm the viability of issuing forecasts are required before that decision could be taken. But the necessary fundamental infrastructure is now in place to enable operational issue of forecasts should this prove desirable.

FIGURE CAPTIONS

1. Part of the six-month forecast for May to October 1992 issued by the Hungarian Meteorological Service. The graphics illustrate changes in temperature (top), likelihood of rain exceeding 1 mm/day (middle) and total rainfall (bottom) over the country from May (Május) to October (Október).

2. Example of a United Kingdom temperature and rainfall monthly forecast. Predictions are given as the most likely quintile for temperature or the most likely tercile for rainfall from periods 1 to 5, 6 to 15 and 16 to 30 days ahead across 10 climatological districts.

3. Example of the preliminary Hungarian forecast layout using five climatological districts. In each group forecasts are presented for average (lower right), maximum and minimum temperatures and for rainfall (lower right). Background hatching, as also used in United Kingdom displays, aids identification of the appropriate quintile/tercile. Anomalies from average/percentages of normal are overprinted. Upper groups indicate predictions for days 1 to 5 using operational products from UKMO (left) and ECMWF (right); lower groups indicate predictions from ensemble means for days 6 to 15 (left) and 16 to 30 (right).

4. Comparison of perfect prognosis skills (i.e. skills obtained from used observed fields) for 15-day specification equations for Hungary and the United Kingdom for equations including and excluding thickness (THK). Results are presented for each month (1 = January); United Kingdom results, originally obtained by half month, have been averaged to give monthly figures. Folland-Painting skill scores (defined in text) are used along the ordinate. a) average temperature; b) rainfall.

5. Average Folland-Painting skill scores for all time ranges for the period since 9 August 1993 when the first Hungarian real-time forecast was made: for (i) average temperature; (ii) rainfall; (iii) maximum temperature; (iv) minimum temperature. Results are provided for United Kingdom issued forecasts (UK_Issued) in addition to forecasts produced without intervention (UK_Auto) from the selected operational model at days 1 to 5 and ensemble mean at other time ranges; the latter are directly comparable to the results for Hungary.

6. Seasonal-average Folland-Painting skill scores at days 1 to 5 for individual seasons: for (i) average temperature; (ii) rainfall; (iii) maximum temperature; (iv) minimum temperature. Results are provided for United Kingdom issued forecasts (UK_Issued) in addition to forecasts produced without intervention from the selected operational model at days 1 to 5 and ensemble mean at other time ranges (UK_Auto); the latter are directly comparable to the results for Hungary. No hindcasts for days 1 to 5 are available for Hungary prior to Autumn 1993.

7. As Figure 6 but for days 6 to 15. Results for Hungary for Spring 1993 were obtained purely as hindcasts; for Summer 1993 there is a mixture of hindcasts and forecasts.

8. As Figure 7 but for days 16 to 30.

9. Time series of Folland-Painting skill scores for Hungarian and issued United Kingdom forecasts at days 1 to 5: for (i) average temperature; (ii) rainfall; (iii) maximum temperature; (iv) minimum temperature. No hindcasts are available for Hungarian forecasts prior to 9 August 1993 (Forecast Number 15). Forecast Number 1 is for 25 January 1993, Number 37 for 13 June 1994. Spring 1993 lies between Forecast Numbers 3 (22 February 1993) and 9 (17 May 1993), Summer 1993 to Forecast Number 15 (23 August 1993), Autumn 1993 to Forecast Number 22 (15 November 1993), Winter 1993/94 to Forecast Number 29 (21 February 1994) and Spring 1994 to Forecast Number 35 (16 May 1994).

10. As Figure 9 but for days 6 to 15 and incorporating Hungarian hindcasts prior to 9 August 1993 (Forecast Number 15).

11. As Figure 10 but for days 16 to 30.

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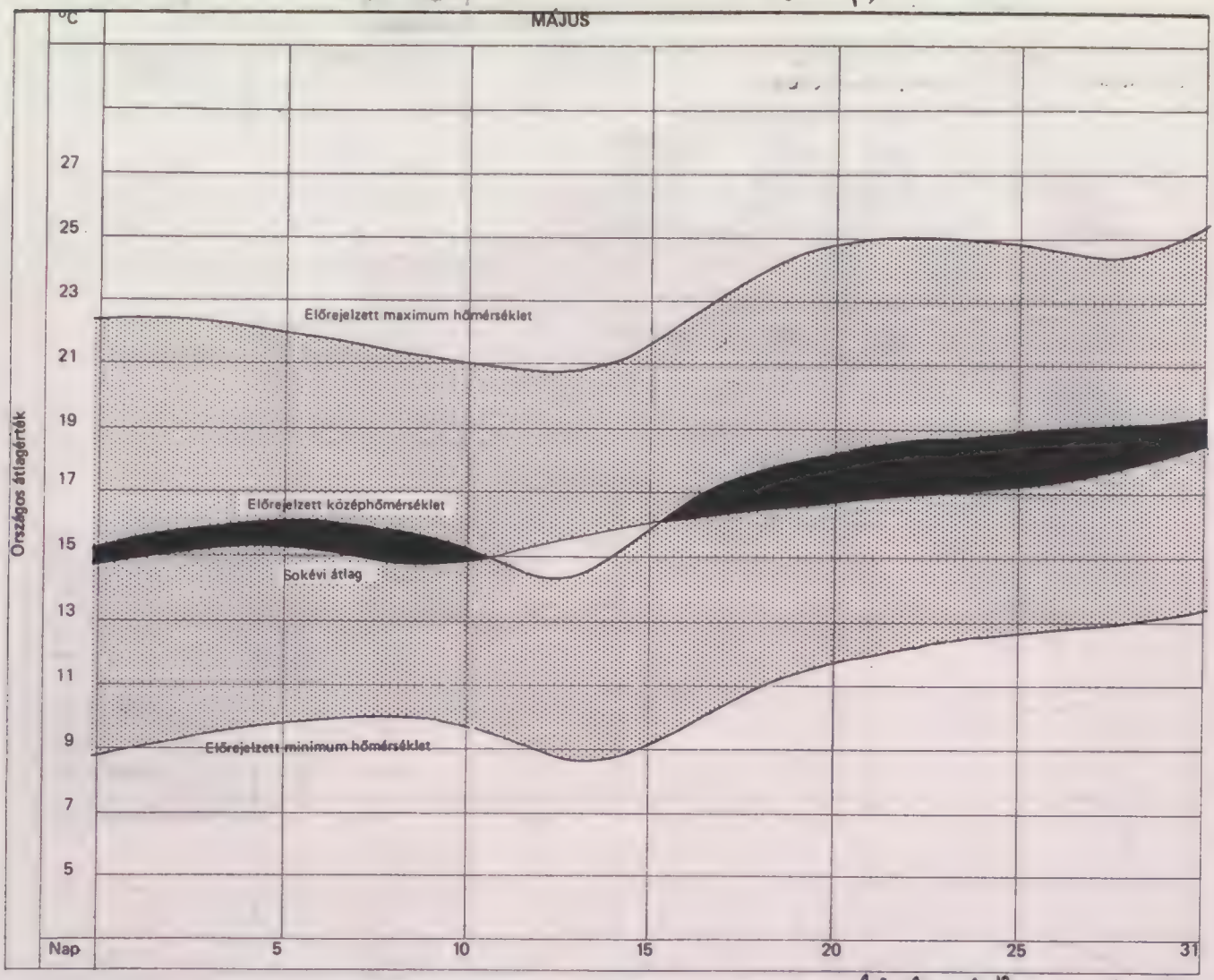
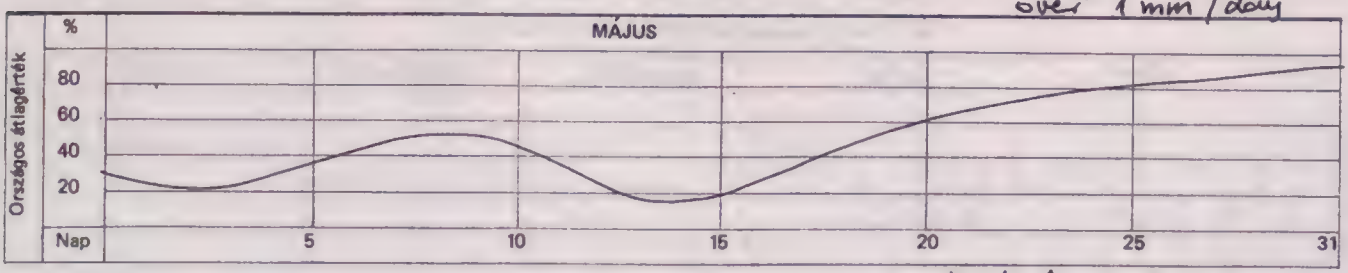
ELŐREJELZÉS

A hőmérséklet várható alakulása (Temp)



A napi 1 mm-t meghaladó csapadék százalékos valószínűsége likelihood of the precip over 1 mm/day



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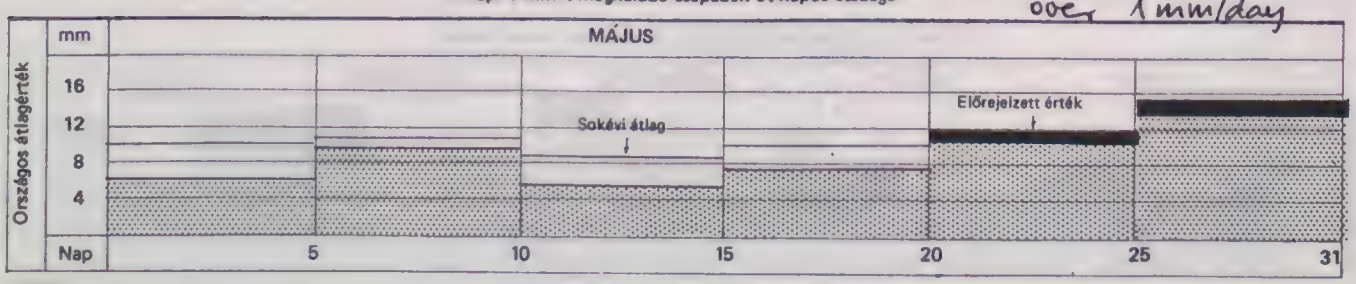


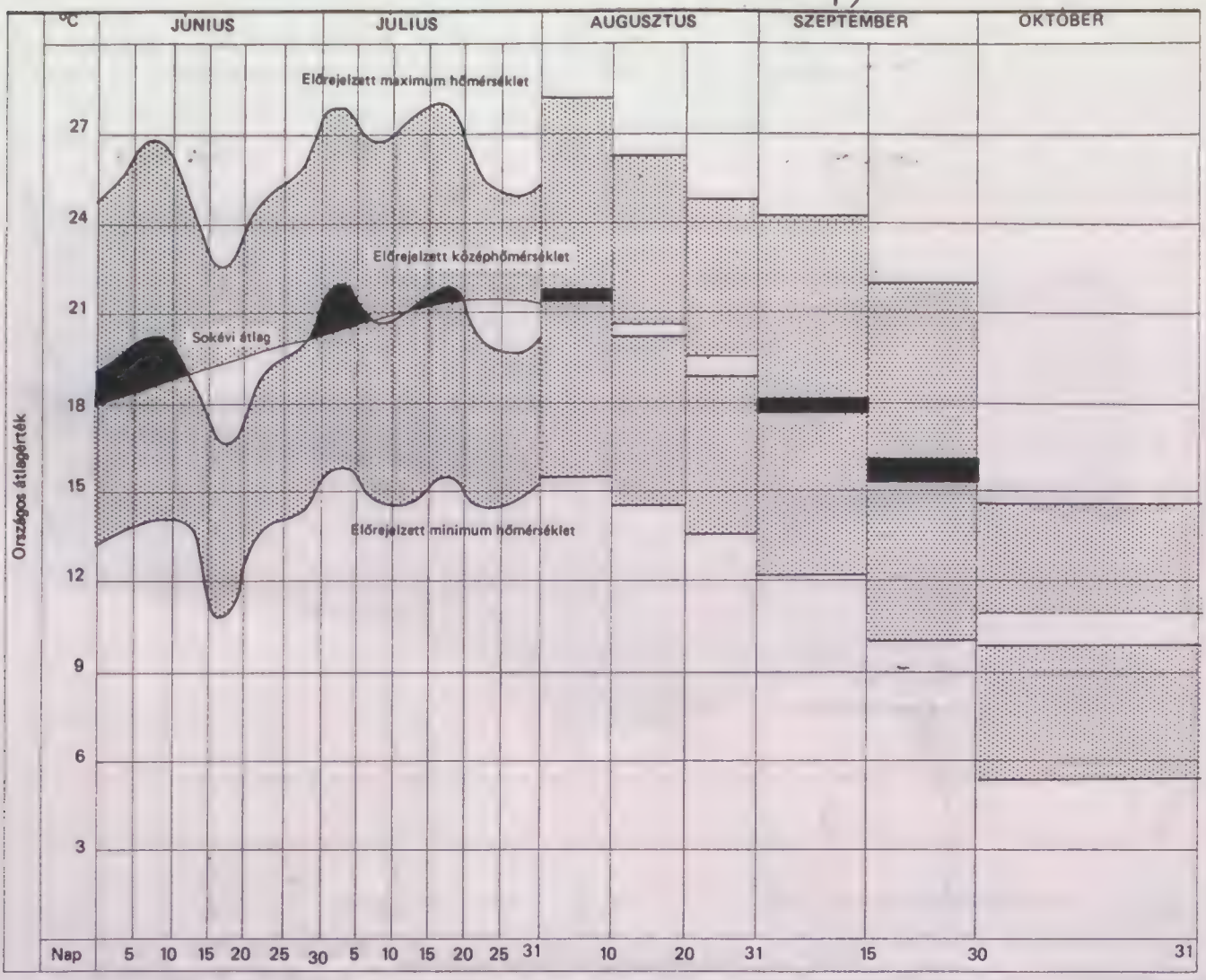
Az előrejelzett érték magasabb, mint a sokévi átlag

Az előrejelzett érték alacsonyabb, mint a sokévi átlag

Figure 1

ELŐREJELZÉS

A hőmérséklet várható alakulása (Temp.)



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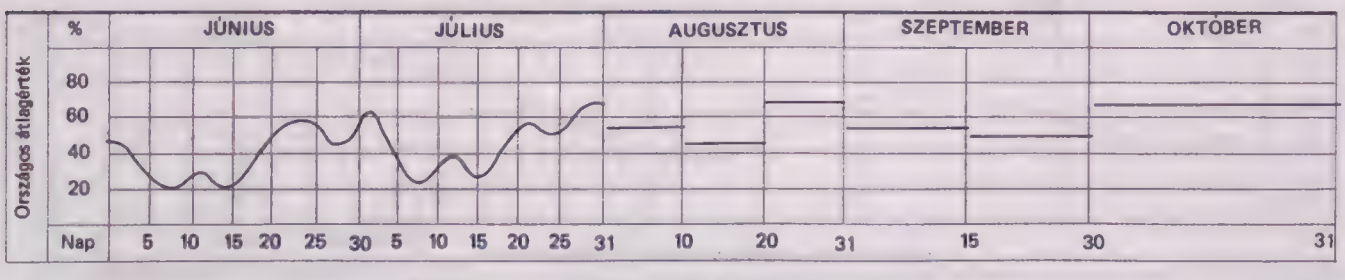

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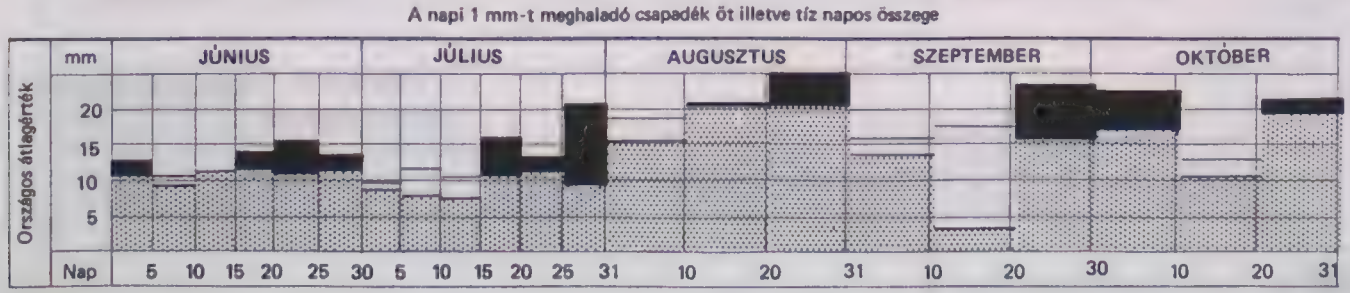
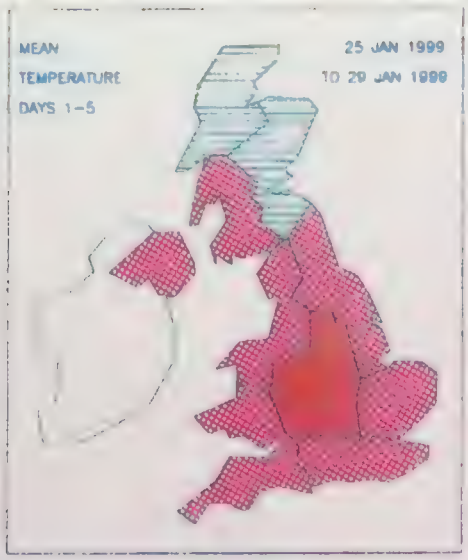


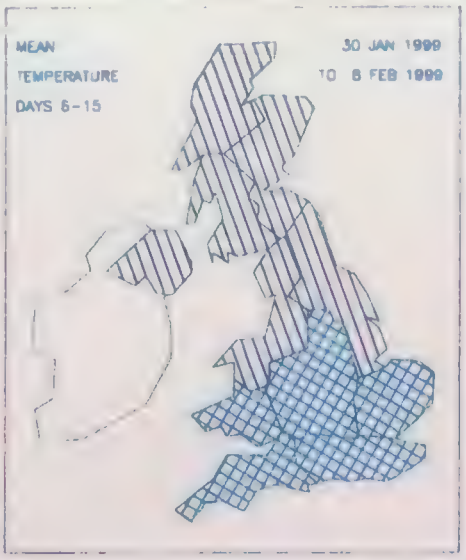
Az előrejelzett érték magasabb, mint a sokévi átlag

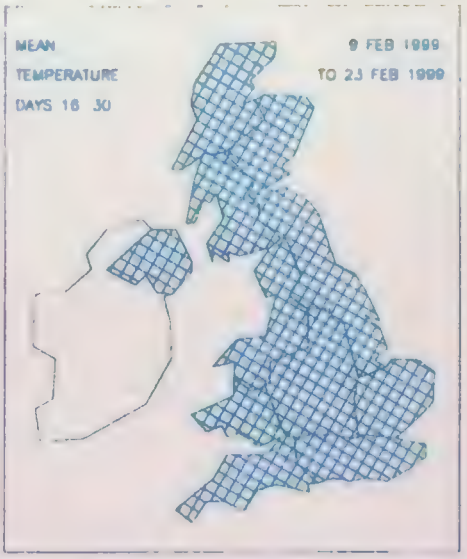
Az előrejelzett érték alacsonyabb, mint a sokévi átlag

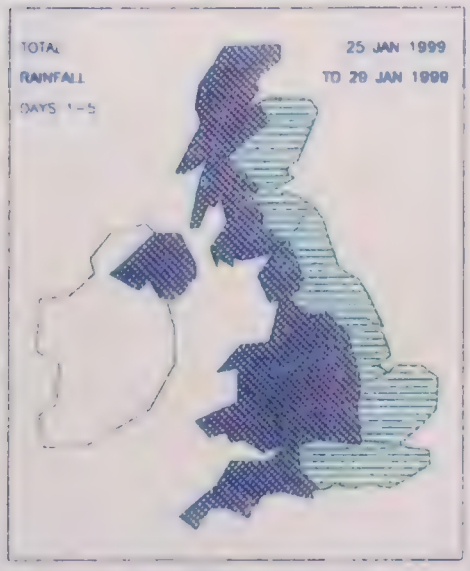
Figure 1 continued

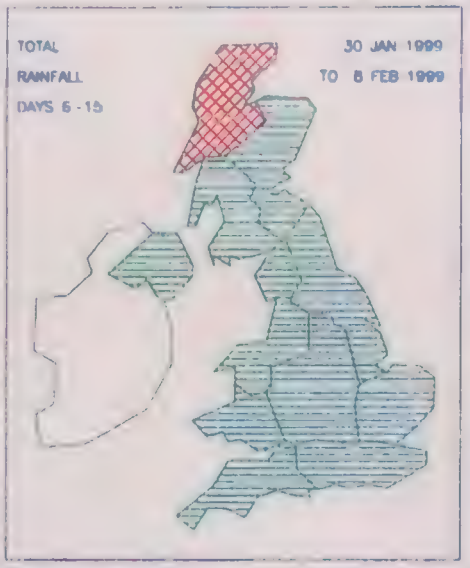
THE MONTHLY PROSPECT

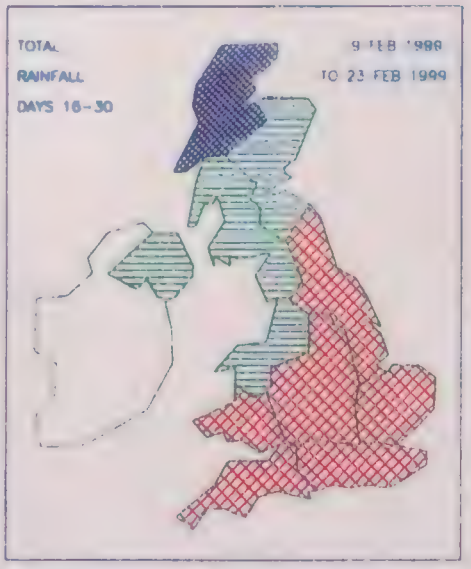












MEAN TEMPERATURE (degC)	LONG TERM AVERAGE FOR 25 JAN 1999 TO 23 FEB 1999
N SCOTLAND	[illegible]
E SCOTLAND	2.5
NE ENGLAND	[illegible]
E ANGLIA	3.4
MIDLANDS	[illegible]
SE ENGLAND	3.8
W SCOTLAND	[illegible]
NW ENGLAND	[illegible]
SW ENGLAND	[illegible]
N.IRELAND	3.4

LEGEND

TEMPERATURE

1 WELL BELOW AVERAGE
2 BELOW AVERAGE
3 ABOUT AVERAGE
4 ABOVE AVERAGE
5 WELL ABOVE AVERAGE

RAINFALL

1 BELOW AVERAGE
2 ABOUT AVERAGE
3 ABOVE AVERAGE

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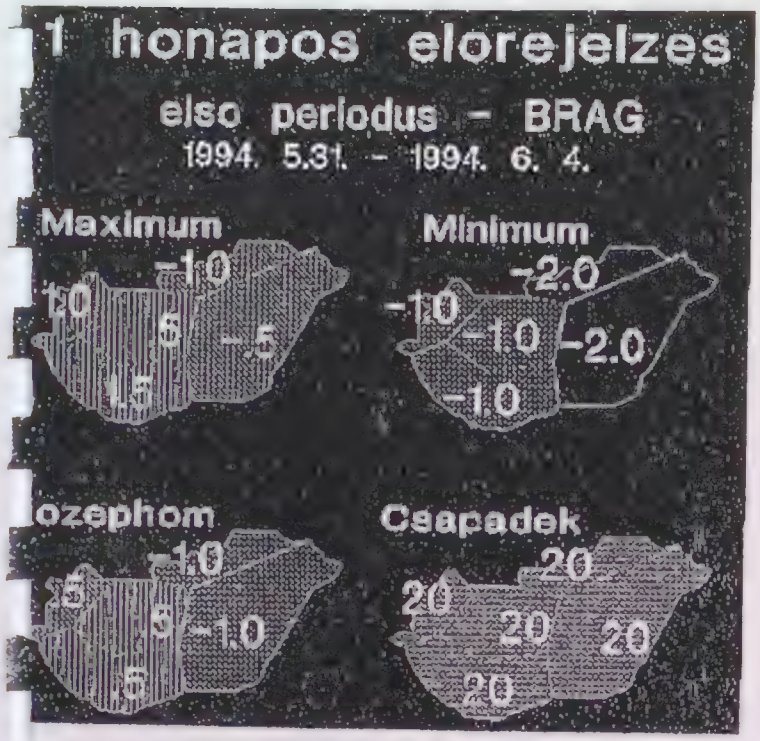
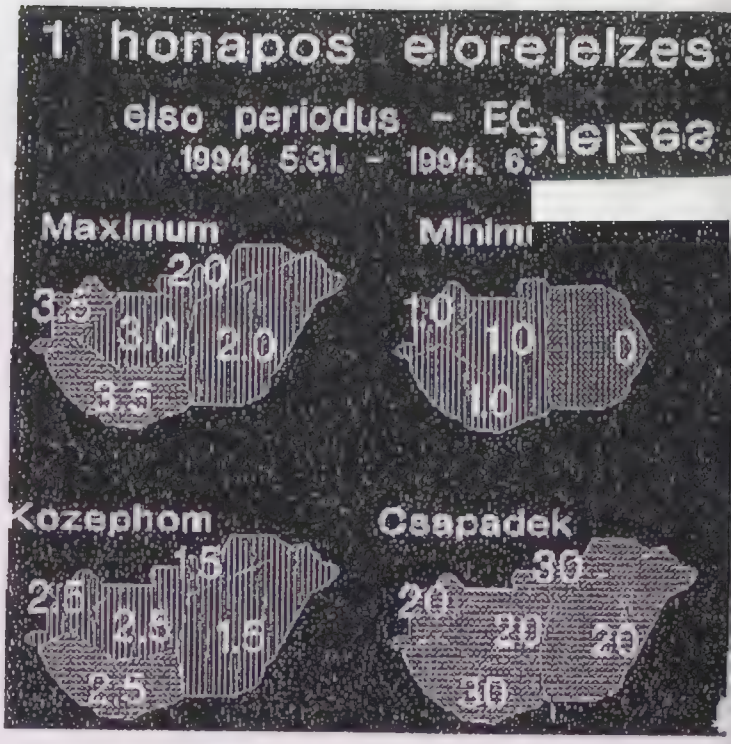
TOTAL RAINFALL (mm)	LONG TERM AVERAGE FOR 25 JAN 1999 TO 23 FEB 1999
N SCOTLAND	85.8
E SCOTLAND	58.3
NE ENGLAND	54.6
E ANGLIA	[illegible]
MIDLANDS	56.2
SE ENGLAND	56.5
W SCOTLAND	[illegible]
NW ENGLAND	[illegible]
SW ENGLAND	[illegible]
N.IRELAND	87.6

Figure 2


1 honapos elorejelzes
elso periodus - BRAG
1994. 5.31. - 1994. 6. 4.
Maximum
Minimum
Csapadek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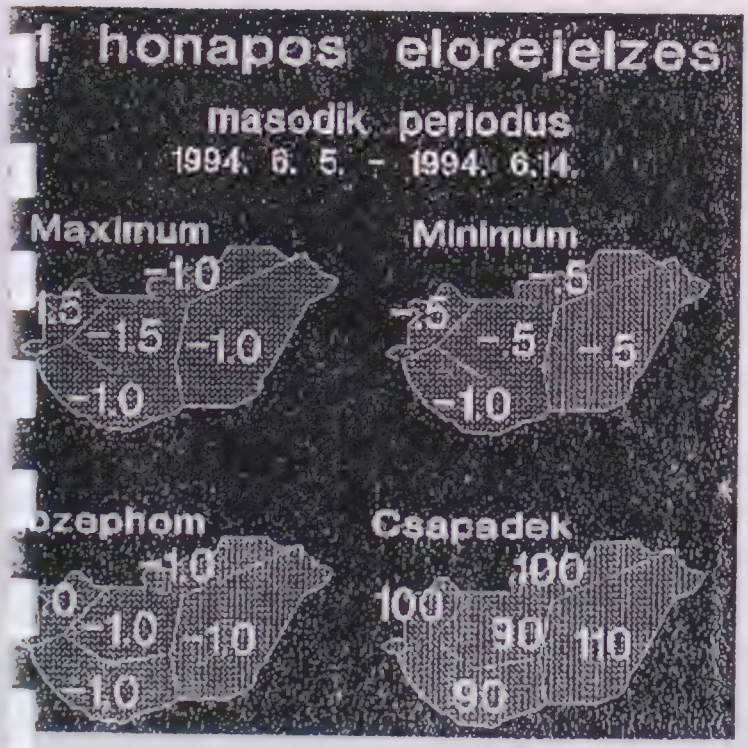
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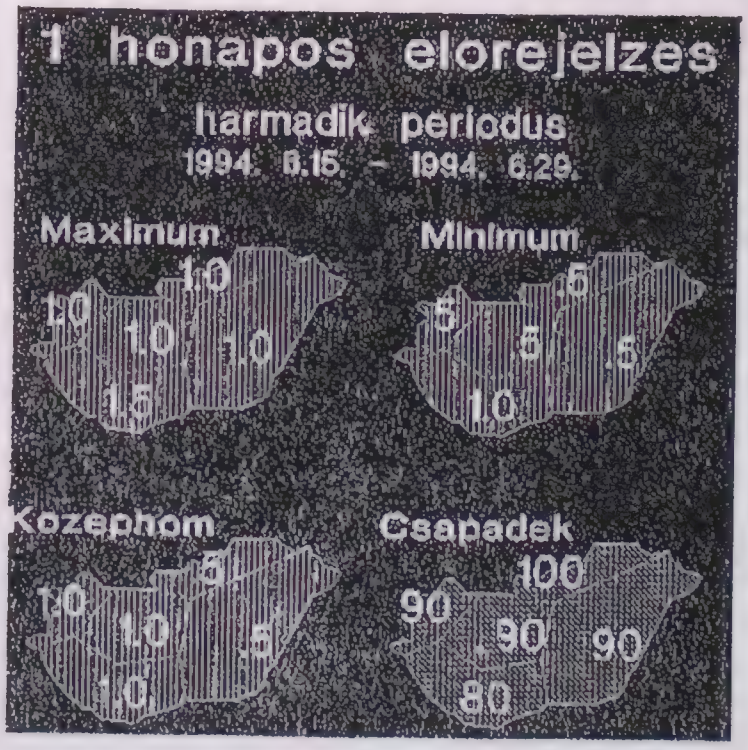
Maximum
Minimum
Csapadek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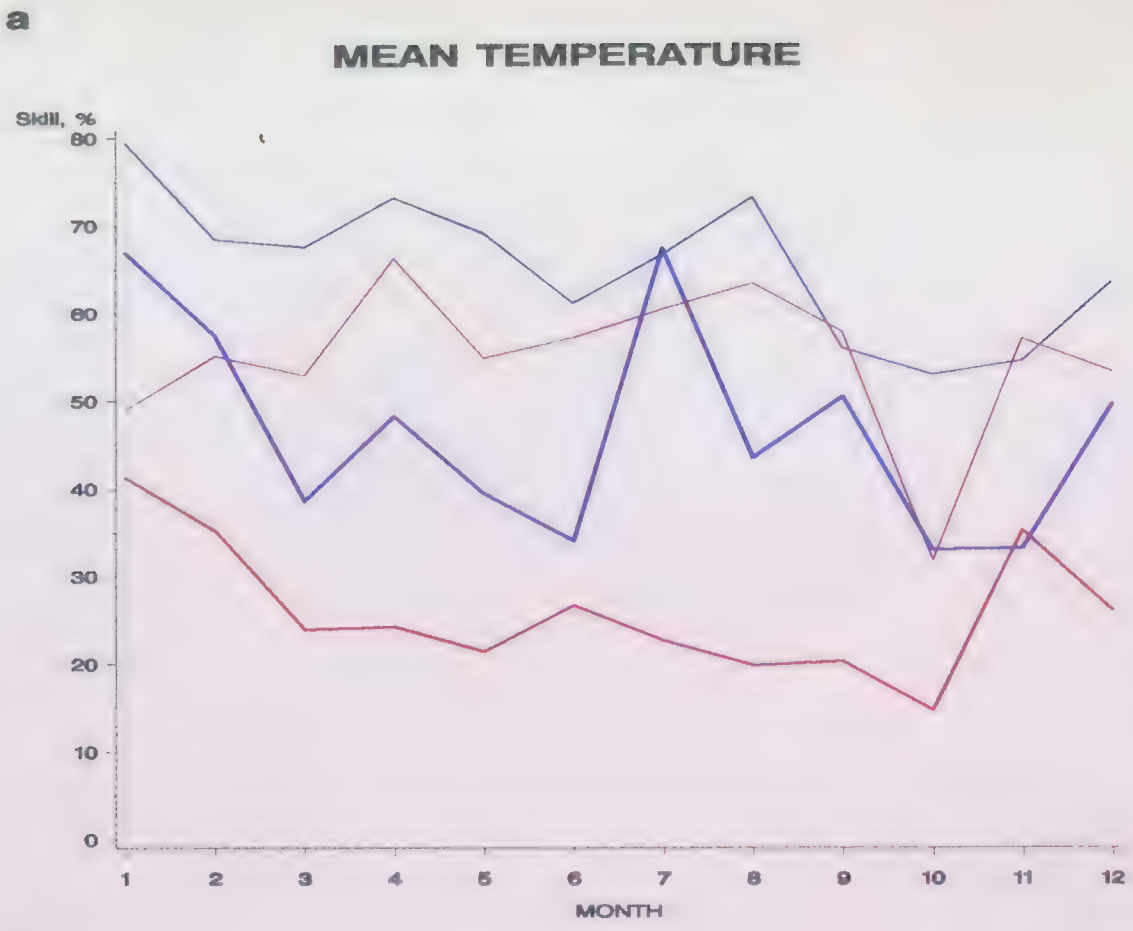
1994. 6.15. - 1994. 6.29.
Maximum
Minimum
Csapadek

Figure 3

a


MEAN TEMPERATURE
Skill, %
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
MONTH

b

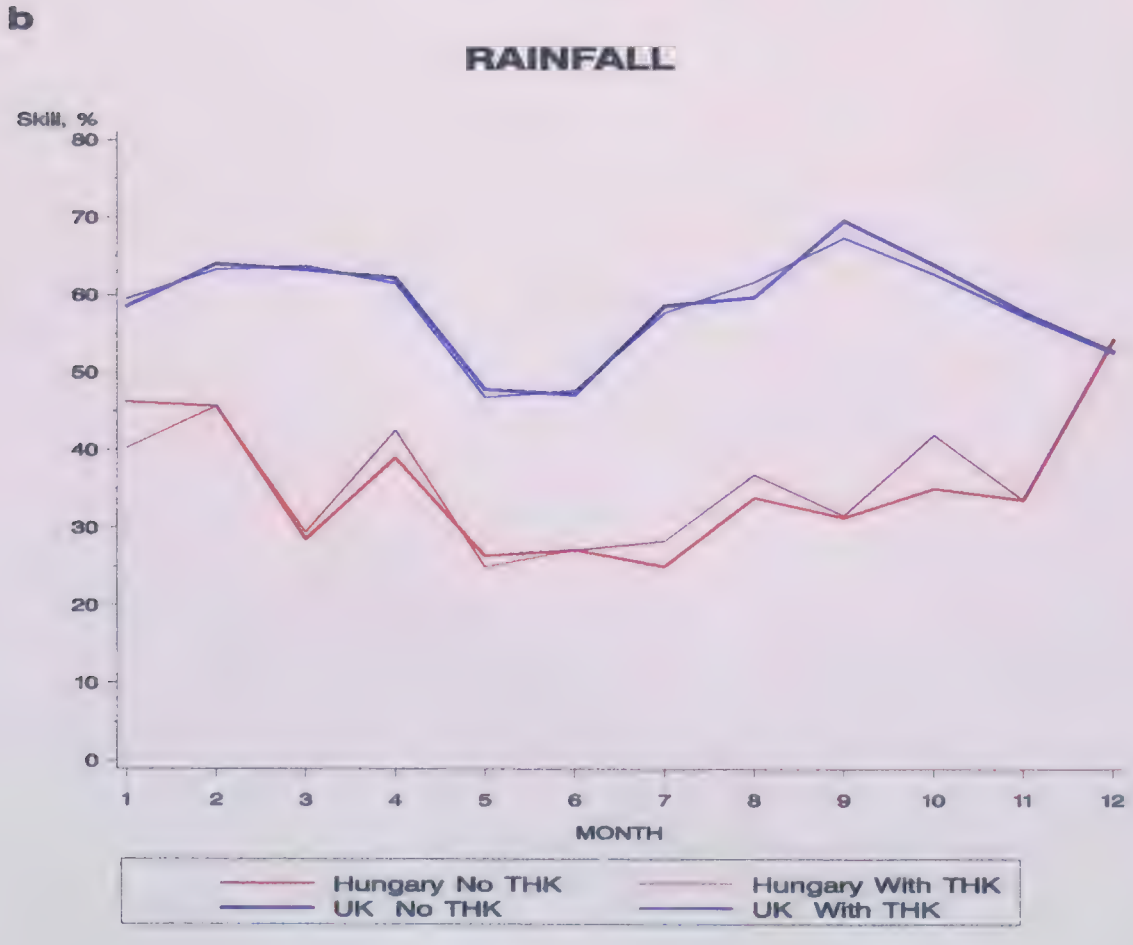
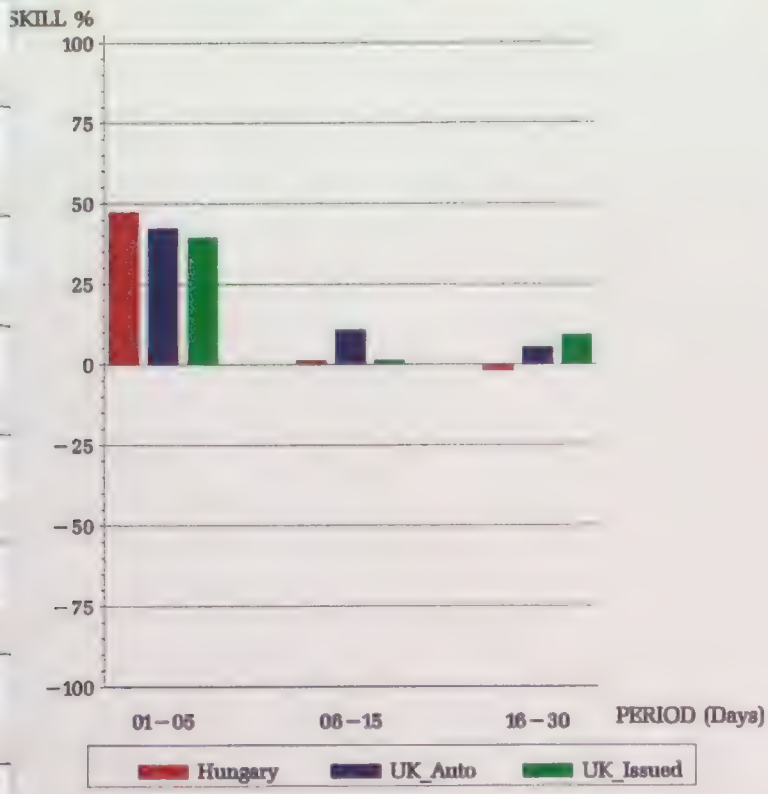
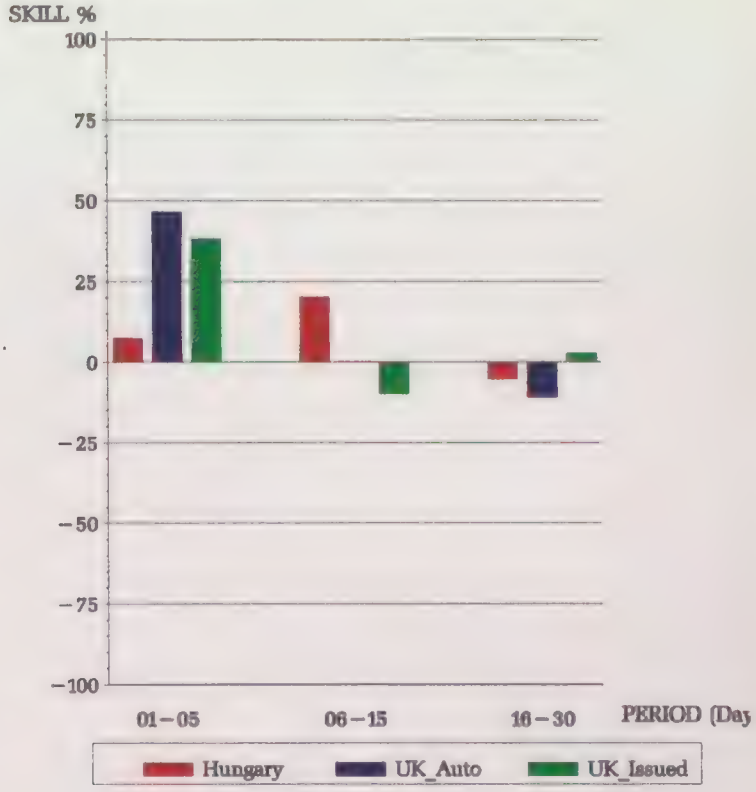

RAINFALL
Skill, %
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
MONTH
Hungary No THK
Hungary With THK
UK No THK
UK With THK

Figure 4

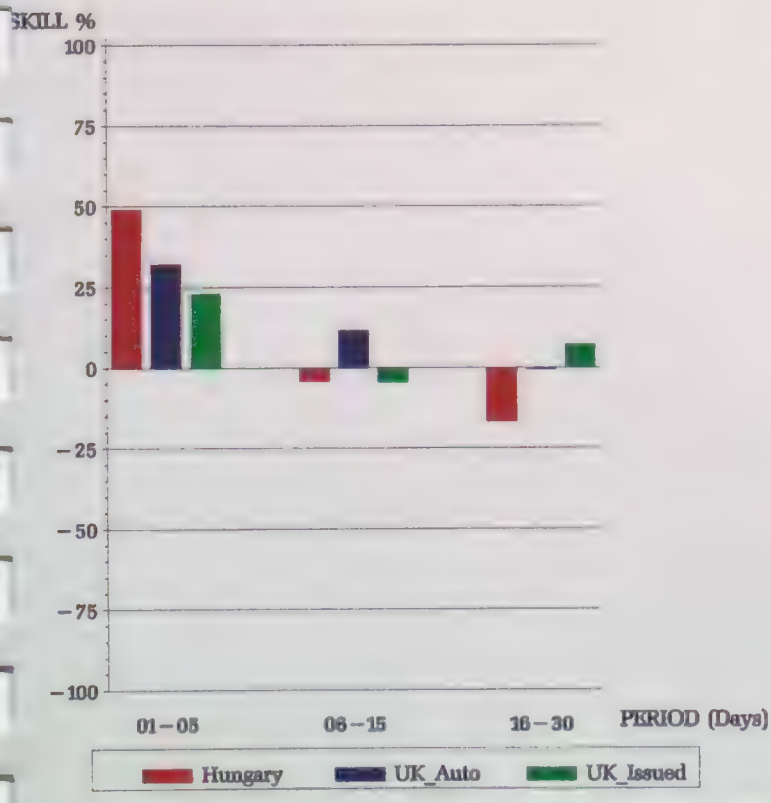
(i) Mean Temperature



(ii) Rainfall



(iii) Max. Temperature



(iv) Min. Temperature

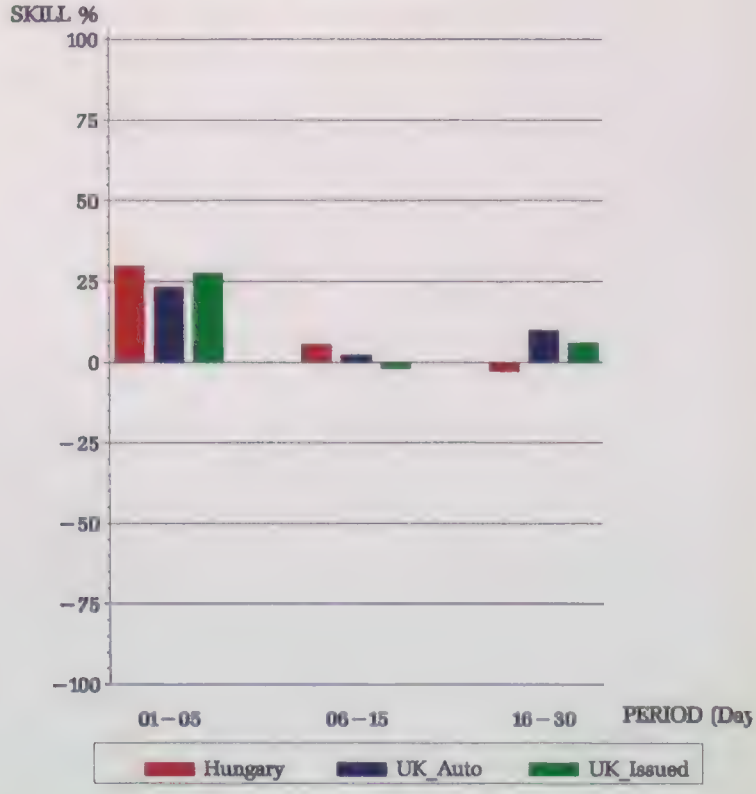
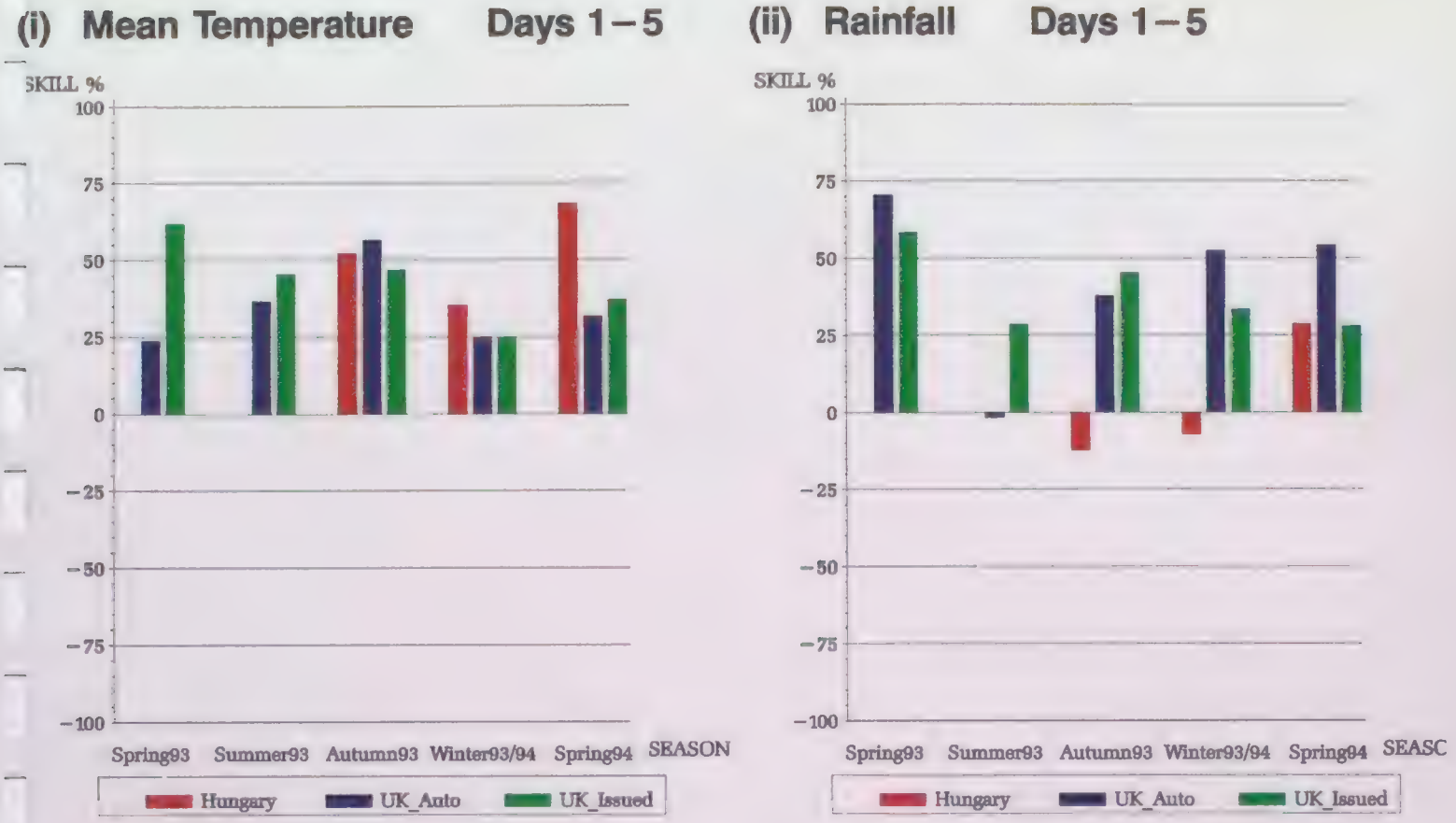


Figure 5


(i) Mean Temperature Days 1–5
SKILL %
100
75
50
25
0
−25
−50
−75
−100
Spring93
Summer93
Autumn93
Winter93/94
Spring94
SEASON
Hungary
UK_Auto
UK_Issued
(ii) Rainfall Days 1–5
SKILL %
100
75
50
25
0
−25
−50
−75
−100
Spring93
Summer93
Autumn93
Winter93/94
Spring94
SEASC
Hungary
UK_Auto
UK_Issued

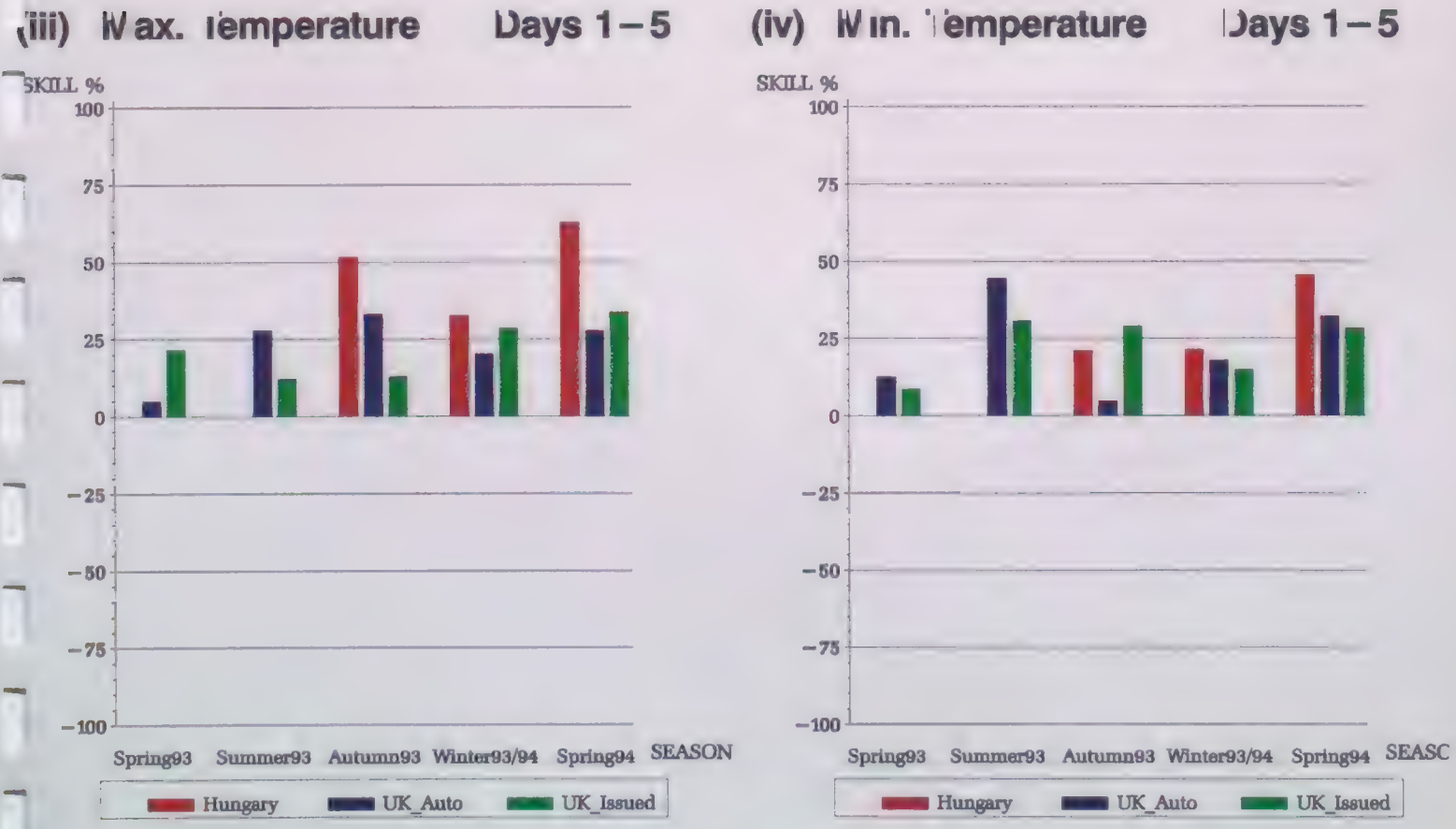
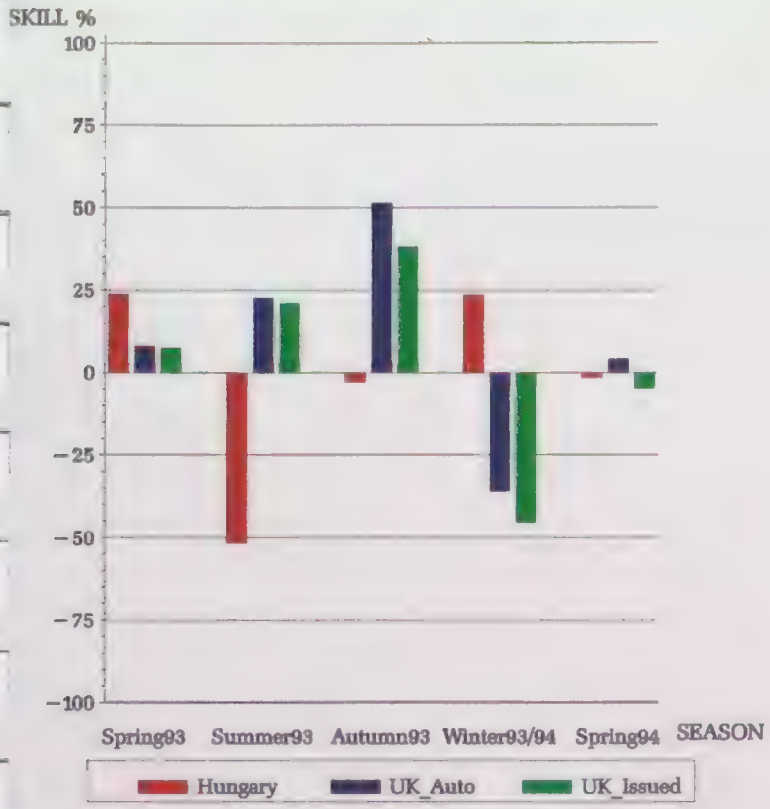

(iii) Max. Temperature Days 1–5
SKILL %
100
75
50
25
0
−25
−50
−75
−100
Spring93
Summer93
Autumn93
Winter93/94
Spring94
SEASON
Hungary
UK_Auto
UK_Issued
(iv) Min. Temperature Days 1–5
SKILL %
100
75
50
25
0
−25
−50
−75
−100
Spring93
Summer93
Autumn93
Winter93/94
Spring94
SEASC
Hungary
UK_Auto
UK_Issued

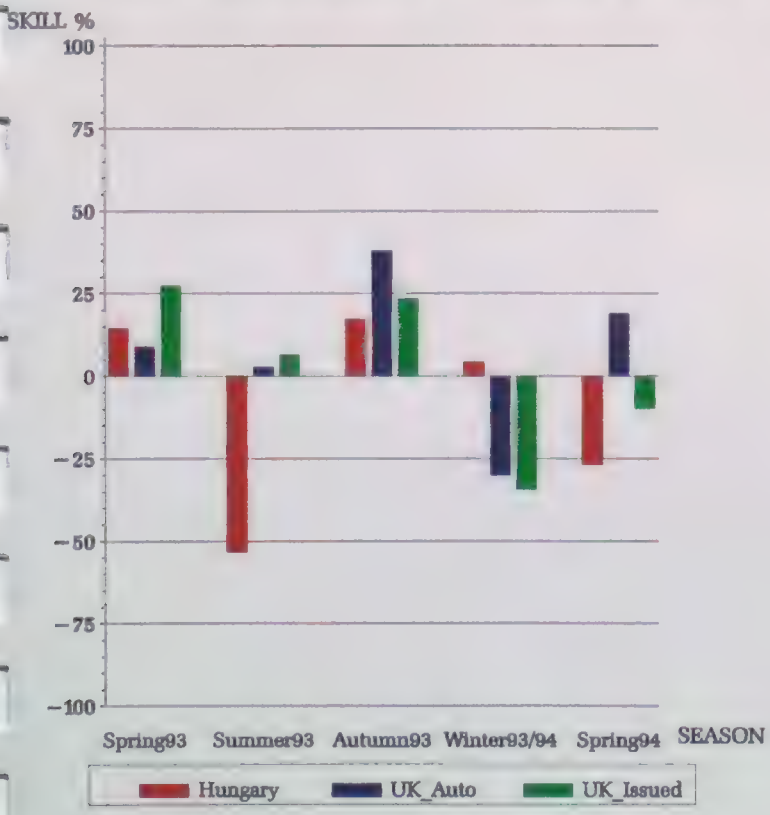
Figure 6

(i) Mean Temperature Days 6–15



(ii) Rainfall Days 6–15

(iii) Max. Temperature Days 6–15



(iv) Min. Temperature Days 6–15

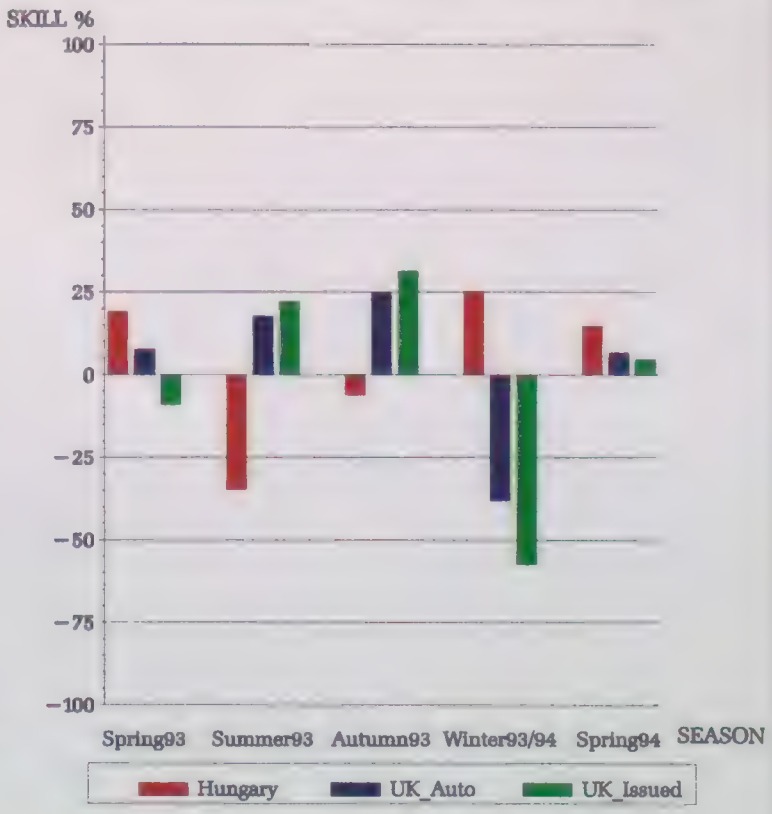
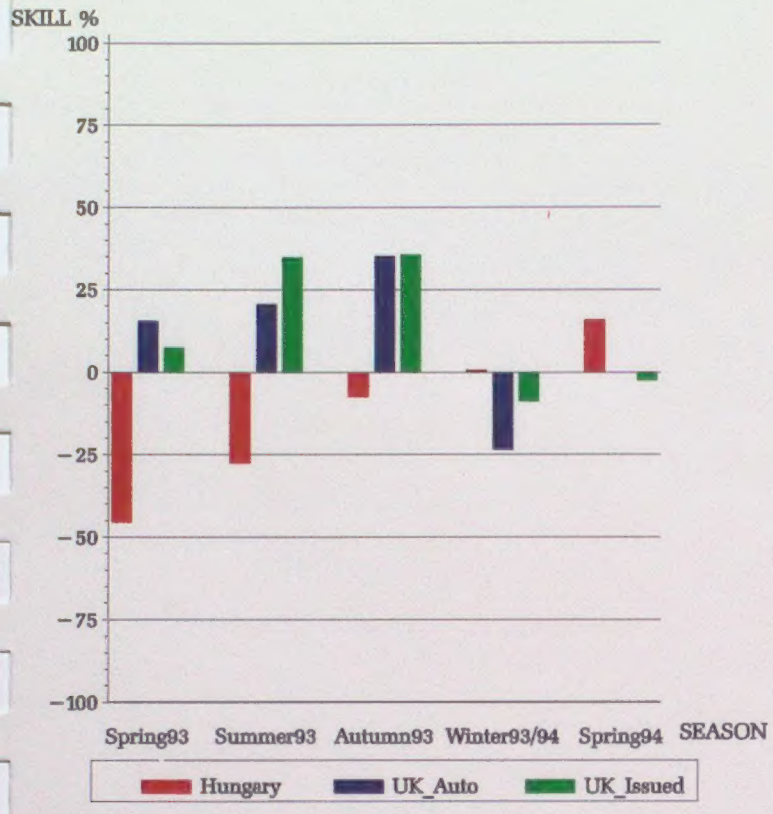
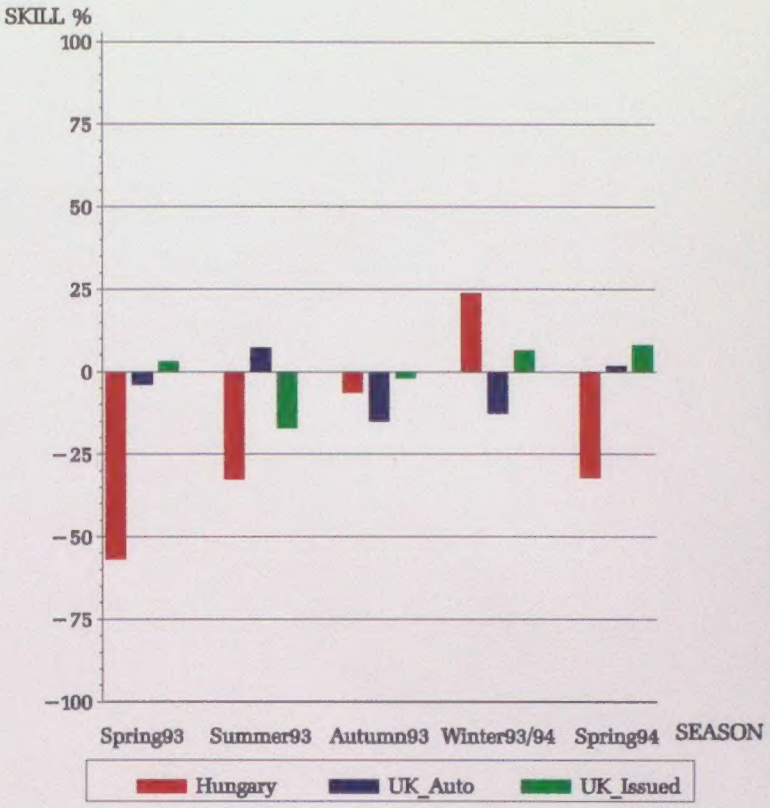


Figure 7

(i) Mean Temperature Days 16 – 30



(ii) Rainfall Days 16 – 30



(iii) Max. Temperature Days 16 – 30

(iv) Min. Temperature Days 16 – 30

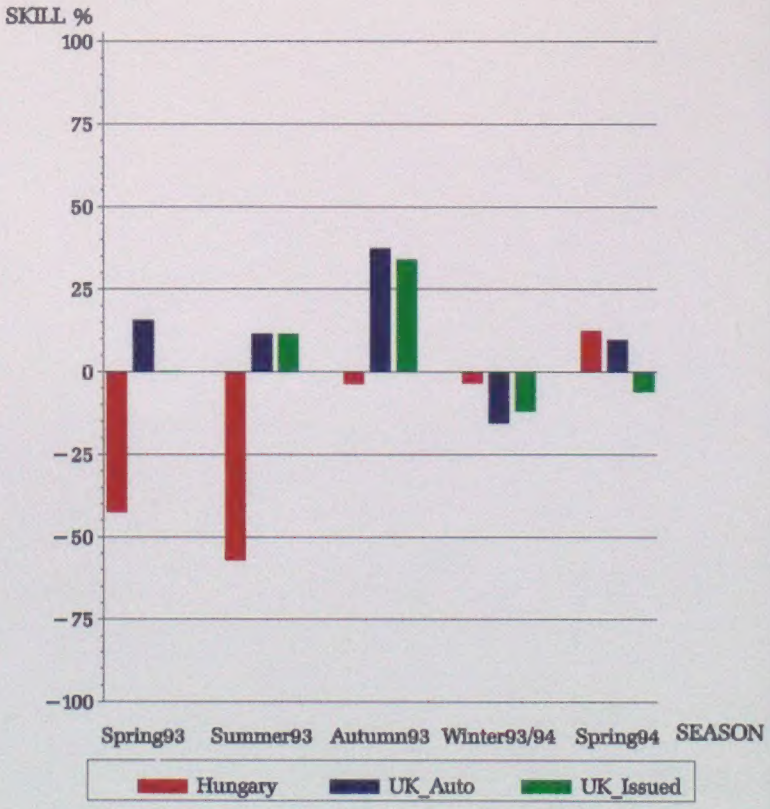
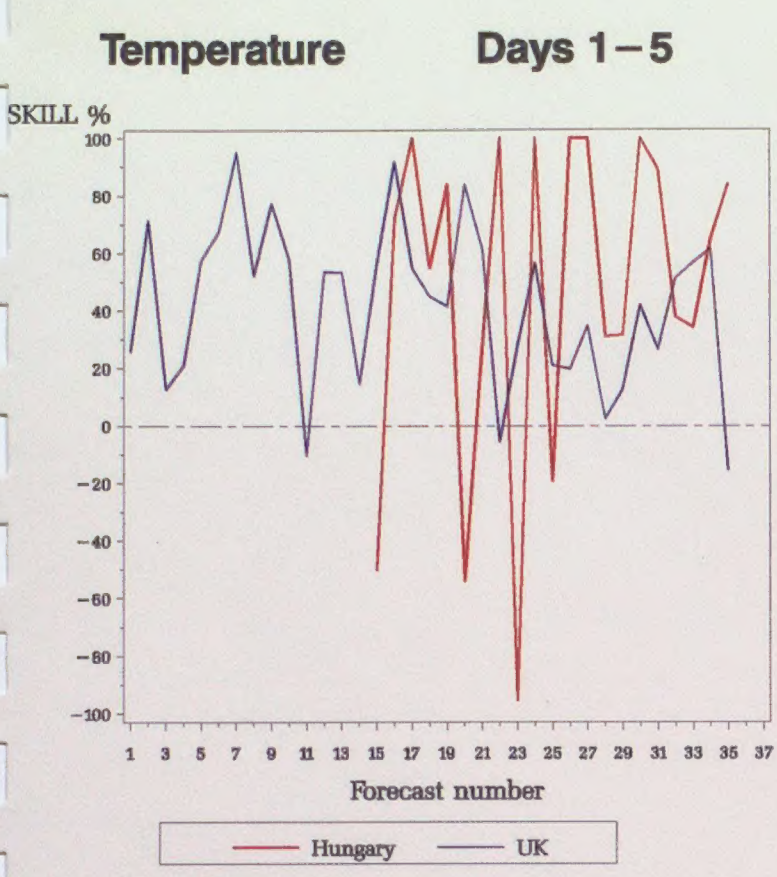
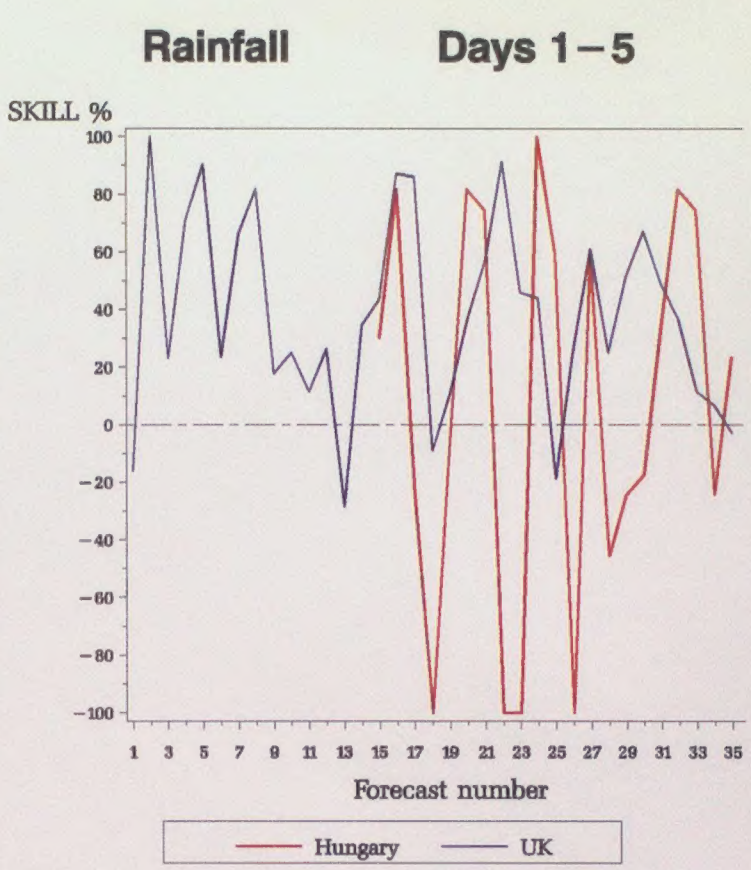


Figure 8

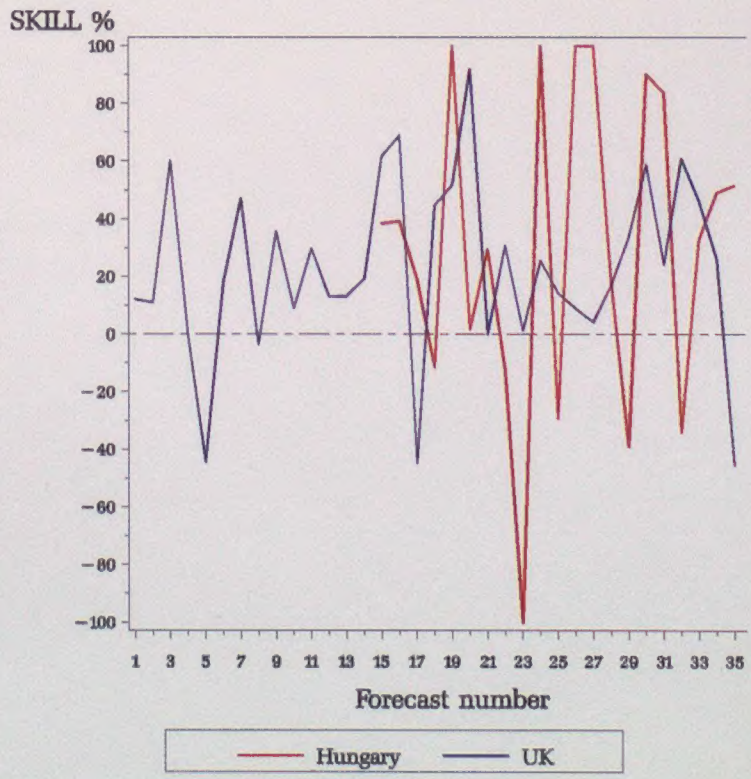


(i)



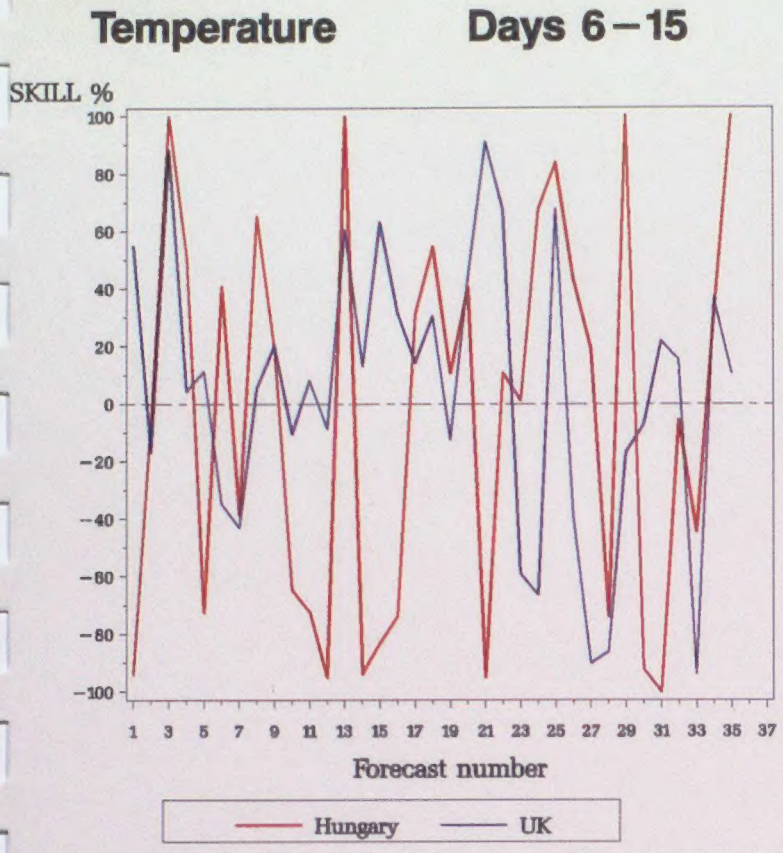
(ii)

(iii)

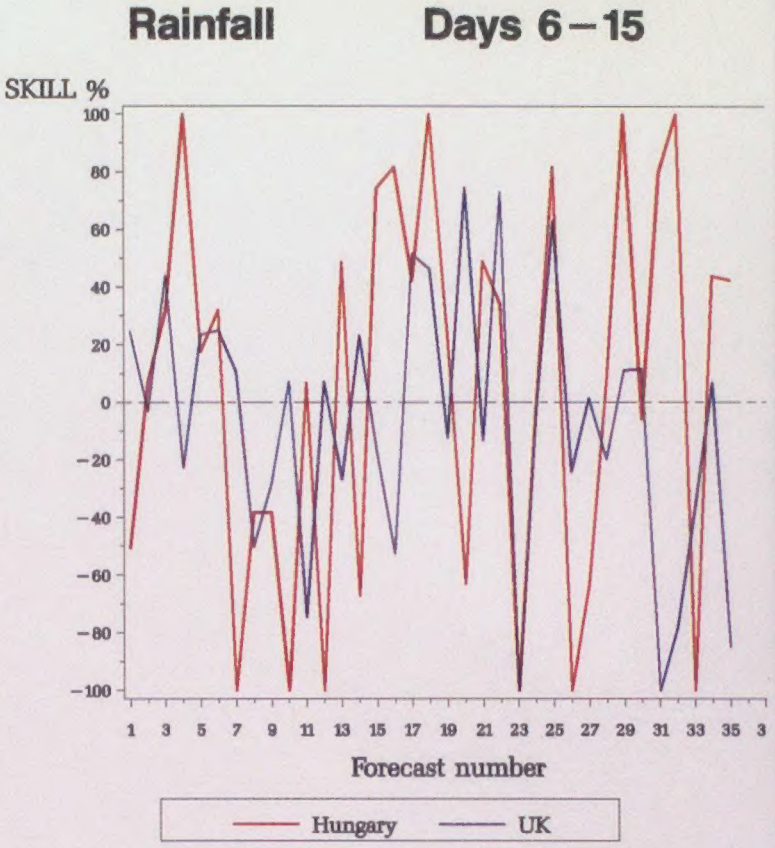


(iv)

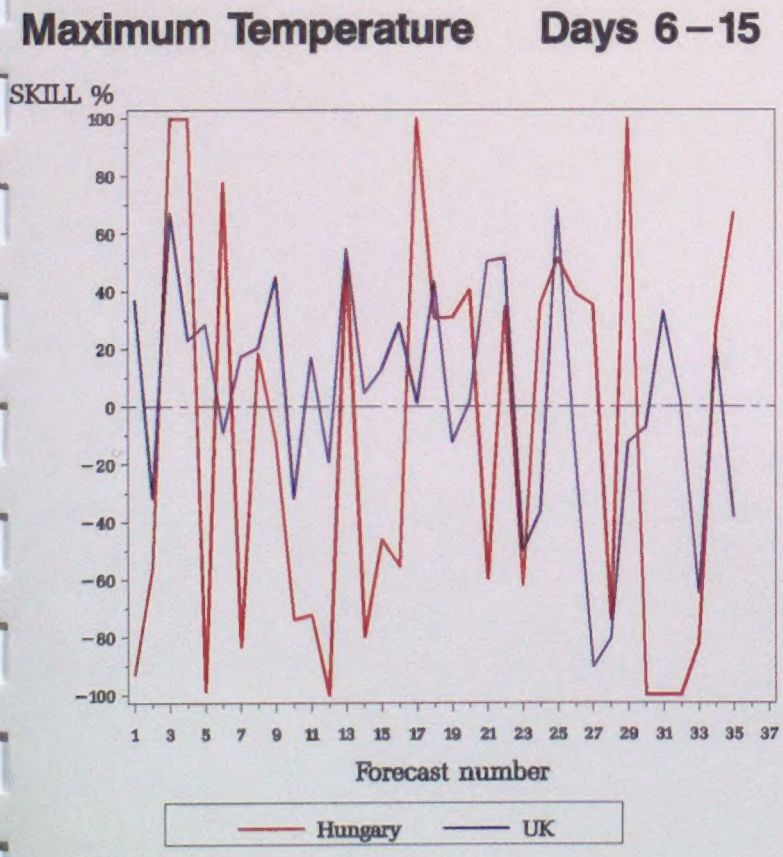
Figure 9



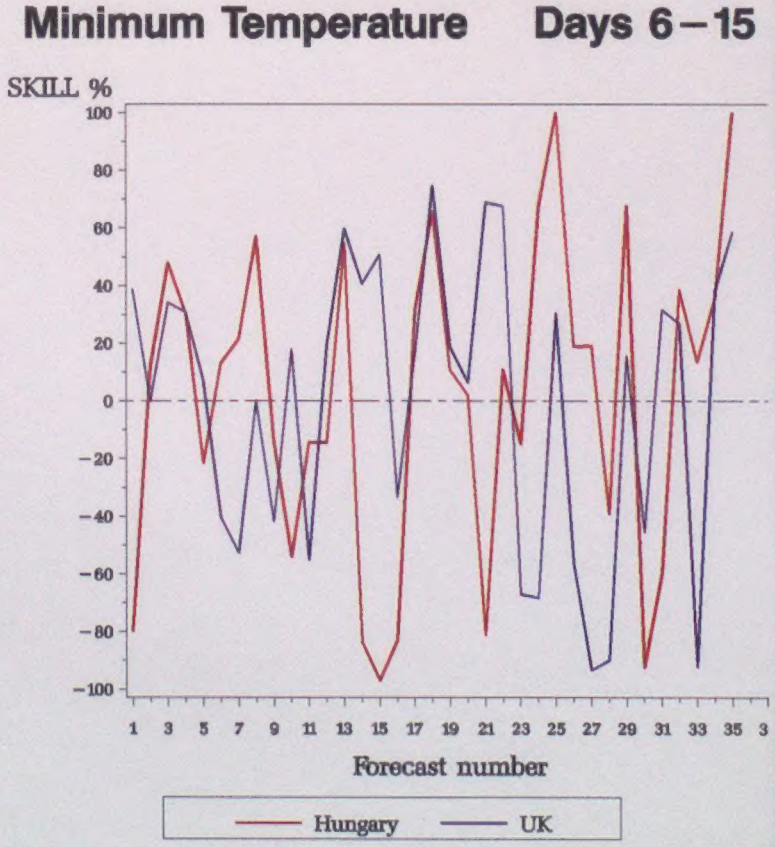
(i)



(ii)

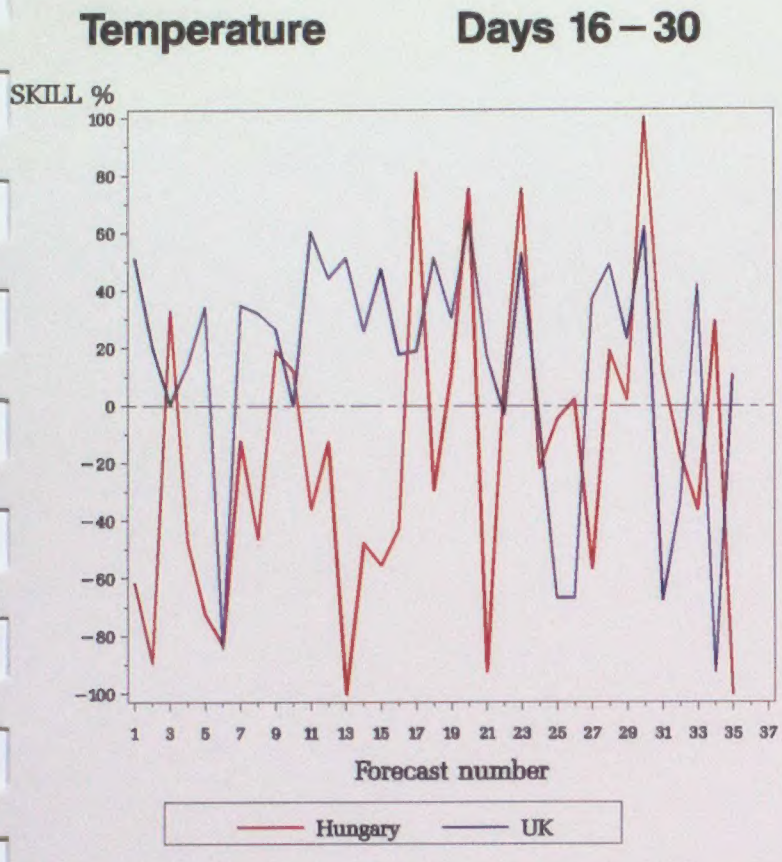


(iii)

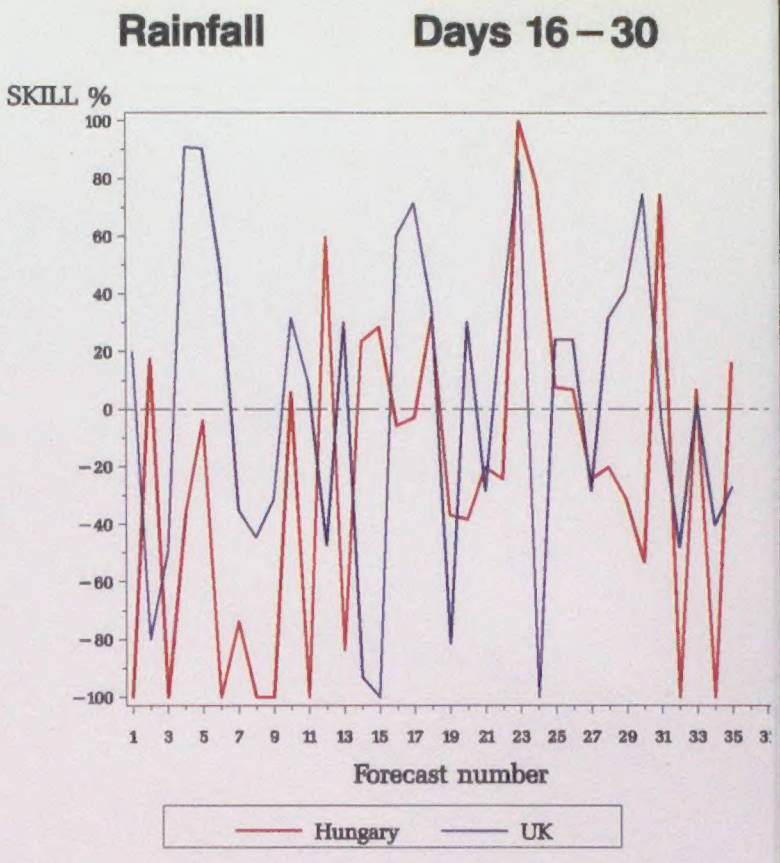


(iv)

Figure 10



(i)



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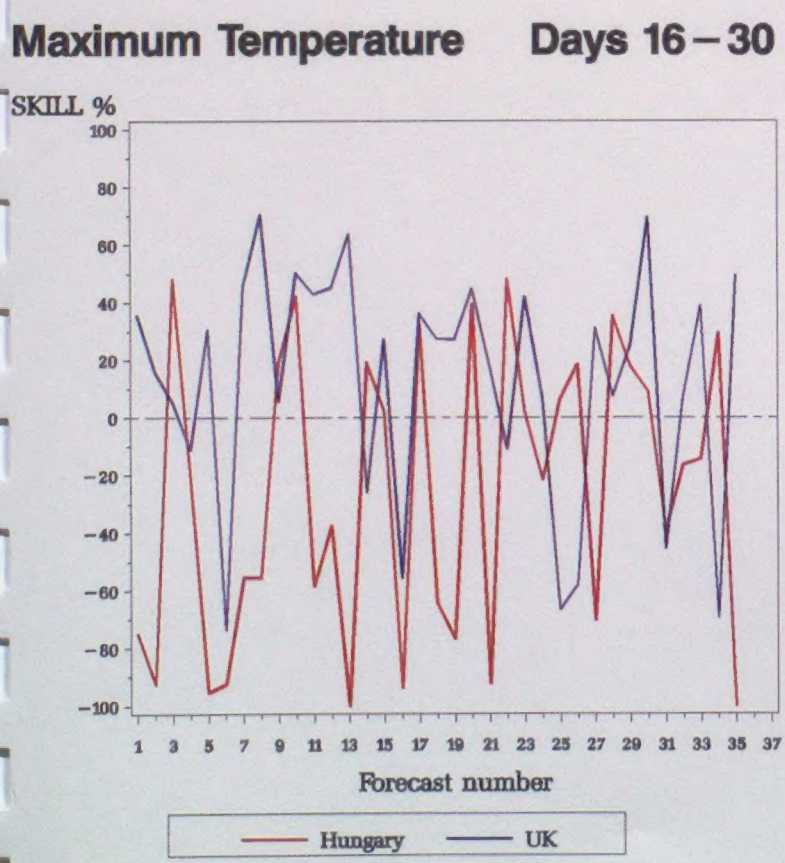
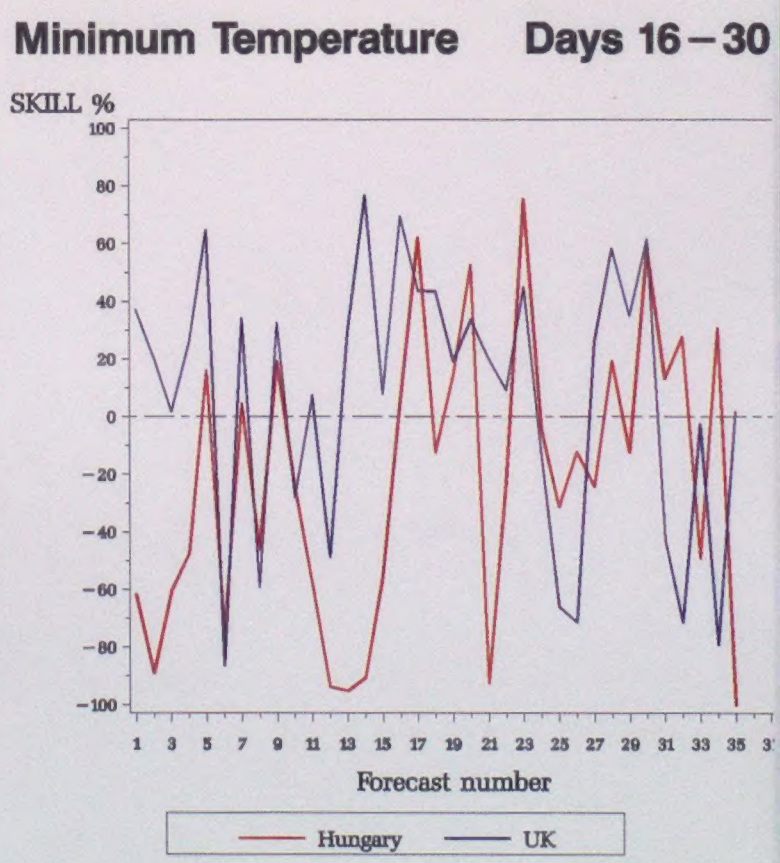


(iii)



(iv)

Figure 11